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Error Analysis in Third-Year Complex Number Topics: A Study of Undergraduate Mathematics Students in a Nigerian University

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Abstract

This study investigates error patterns made by undergraduate mathematics education students at a Nigerian university when solving problems on third-year complex number topics. Eighteen third-year students enrolled in MATH307 (Complex Analysis I) participated in the study. Participants attempted six open-ended tasks covering algebraic simplification, the complex conjugate, division, multiplication, modulus and argument of a complex quotient, and the application of De Moivre's theorem. Their handwritten responses were analysed using a four-category error taxonomy: Conceptual, Interpretation, Procedural, and Technical. Findings reveal that all four error types were present across the six tasks, with Conceptual errors being the most frequent (n = 26), driven largely by near-total failure on the De Moivre's theorem task (12 of 18 students). Interpretation errors (n = 17) exhibited a consistent cross-task pattern: students systematically substituted modulus-argument or polar-form procedures for tasks requiring simpler operations, reflecting overgeneralization of a recently acquired schema. Procedural errors (n = 7) were concentrated in multiplication and division tasks, while Technical errors (n = 3) involved arithmetic lapses in otherwise structurally correct solutions. A key cross-task finding is the cumulative dependency of errors: students who could not divide complex numbers in Question 3 were subsequently unable to simplify the quotient required by Question 5. These findings carry specific implications for the sequencing and emphasis of instruction in complex number courses.

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Introduction

Complex numbers occupy a pivotal position in undergraduate mathematics: they extend the real number system, underpin vast areas of applied mathematics and physics, and demand from students a qualitatively different mode of algebraic reasoning. Yet they also represent one of the most conceptually demanding topics that first-encounter university students face. The equation $x^2 + 1 = 0$ has no solution in the real number system, and its resolution through the introduction of the imaginary unit i defined by $i^2 = -1$, and the consequent definition of a complex number $z = a + bi$ where $a, b \in \mathbb{R}$ requires students to fundamentally revise their understanding of what a 'number' is (Caglayan, 2016; Nordlander & Nordlander, 2012; Smith et al., 2015).

At secondary (high) school, students typically learn that $x^2 + 1 = 0$ has 'no real roots' and move on. University study requires them to dismantle this conclusion and reconstruct their number concept to accommodate the complex system. This cognitive reconstruction is not automatic, and research consistently shows that students encounter difficulty not only in performing calculations with complex numbers, but in understanding what the various representations of complex numbers mean, how they relate to one another, and which representation is appropriate for which task (Danenhower, 2006; Panaoura et al., 2006; Soto-Johnson & Troup, 2014).

Error analysis i.e., the systematic identification, categorization, and interpretation of students' mistakes offers a principled method for making these difficulties visible and pedagogically tractable (Kiat, 2005; Orton, 1983; Siyepu, 2015). Rather than treating wrong answers as mere evidence of failure, error analysis reads them as data: traces of the reasoning processes, incomplete schemas, and procedural gaps that underlie students' mathematical thinking. Despite the well-documented difficulties with complex numbers, few studies have subjected students' errors in this domain to systematic qualitative analysis.

While error analysis has been applied productively to calculus, integration, and trigonometric differentiation, few studies have systematically categorised errors in introductory complex numbers and fewer still have done so in sub-Saharan African university contexts, where instructional resources, class sizes, and prior student preparation differ markedly from the Western settings in which most of this literature was produced. This study addresses that gap. We present a question-by-question error analysis of 18 undergraduate mathematics education students' responses to six complex number tasks, with each error type grounded in and illustrated by the students' own handwritten work. The analysis is guided by the following research question:

Research Questions:

1. What types of errors do undergraduate mathematics education students make when solving introductory complex number problems, and what do these errors reveal about the conceptual and procedural gaps in their understanding?
2. How do error types and frequencies differ across tasks of varying conceptual demand?

Literature Review

Students' Difficulties with Complex Numbers

Research consistently identifies three interrelated clusters of difficulty in complex number learning: performing algebraic calculations, converting between representational forms (rectangular, polar, exponential), and selecting the appropriate form for a given problem (Smith et al., 2015). These clusters are not independent. A task requiring the modulus and argument of a complex quotient, for example, demands prior competence in division and any failure in division propagates forward into the modulus-argument calculation. The cumulative structure of complex number knowledge means that early conceptual gaps have compounding downstream effects.

Nordlander and Nordlander (2012) attribute many student difficulties to an impoverished concept image of complex numbers. For example, students may be able to manipulate complex expressions procedurally without having a coherent mental model of what a complex number is or what its representations mean geometrically. They recommend visual and multi-representational instruction to build stable concept images before procedural work begins. Danenhower (2006), working within the APOS (Action-Process-Object-Schema) framework, found that university students could often perform operations in polar and rectangular forms in isolation but could not coordinate or translate between them. This finding is directly relevant to tasks involving De Moivre's theorem.

Panaoura et al. (2006) demonstrated that students' performance varied markedly with representational form — competence in one form did not transfer automatically to another while Soto-Johnson and Troup (2014) found that students' geometric reasoning about complex multiplication was fragile and heavily context-dependent. Caglayan (2016) similarly found that even mathematics teachers struggled with complex number visualization, particularly in dynamic geometry environments. This body of work establishes that difficulty with complex numbers is not merely a matter of insufficient practice: it reflects deep structural challenges in building and coordinating multiple representations of a genuinely abstract mathematical object.

More recent scholarship confirms the persistence of these difficulties. Seloane et al. (2023) found that undergraduate engineering mathematics students at a South African university experienced significant conceptual and procedural obstacles with complex numbers, particularly when required to coordinate between rectangular and polar representations. Their study demonstrated that GeoGebra-enriched activities providing representational support yielded meaningful improvements in both conceptual and procedural knowledge and suggest that visual, technology-mediated approaches can address the representational coordination difficulties identified in earlier work. Similarly, Gaona et al. (2022), studying prospective mathematics teachers in initial teacher training, found that students' previous mathematical knowledge was insufficiently solid to allow them to interpret feedback from computer algebra systems during complex number tasks. This finding extends the difficulty literature beyond undergraduate students to include those preparing to teach the topic.

Error Analysis as a Methodological and Theoretical Framework

Error analysis treats students' mistakes not as noise to be filtered out but as signal i.e., informative traces of the cognitive structures, schemas, and gaps that shape mathematical reasoning (Kiat, 2005; Siyepu, 2015). Applied as a methodological tool, it enables researchers to move beyond binary correct/incorrect scoring to identify the specific cognitive source of each error. Orton (1983) used it to reveal that students' errors in differentiation reflected specific structural misunderstandings rather than random lapses; Kiat (2005) demonstrated that integration errors clustered systematically into procedural and interpretive types; and Siyepu (2015) showed that errors in trigonometric differentiation were predominantly conceptual, with clear instructional implications.

Each study demonstrated that error patterns are systematic rather than random, and that identifying them precisely yields actionable implications for instruction.

This study adopts the four-category error taxonomy from Siyepu (2015) and Kiat (2005), which extends and operationalises Olivier's (2003) foundational categories of conceptual and procedural errors. The taxonomy distinguishes Conceptual, Interpretation, Procedural, and Technical errors — each capturing a qualitatively distinct proximal cause of mathematical error and its diagnostic power lies not in any single category but in the pattern of categories across tasks. The operational definitions of each category are set out in the Conceptual Framework section below.

Pedagogical Responses

Karakok et al. (2015) argued for a carefully sequenced introduction to complex numbers, beginning with the identity $i = \sqrt{-1}$ before introducing algebraic operations which is a sequencing that builds from the foundational to the complex. They found that even experienced secondary teachers lacked structural understanding of certain complex number forms and could not fluently transition between representations, underscoring the importance of carefully sequenced instruction that builds from simpler to more complex forms. Ahmad and Shahrill (2014) emphasized deep content knowledge on the part of instructors and explicit attention to the commutative and associative laws governing both real and imaginary parts. Yatab and Shahrill (2014) advocated multimedia-based instruction for abstract topics, while Matzin et al. (2013) highlighted the role of current, appropriate textbooks. McCarthy and Smuts (1997) and Congos and Schoeps (1993) provided evidence for the effectiveness of supplementary instruction and peer learning sessions in supporting students through difficult mathematical material.

Conceptual Framework

While the error taxonomy adopted here is described in methodological terms in the Literature Review, this section sets out the precise operational definitions applied in the analysis of student responses in the present study. This study adopted error analysis as both its theoretical lens and its analytical method. Following Siyepu (2015), Kiat (2005), and Olivier (2003), we employ a four-category taxonomy:

1. Conceptual error: An error rooted in a fundamental misunderstanding of a mathematical concept or the relationships between concepts. The student has an incorrect or incomplete schema for the relevant idea not merely a procedural lapse.
2. Interpretation error: An error arising when a student misidentifies what a task is asking typically through overgeneralization of a related procedure to a context where it does not apply. The student may execute a procedure correctly, but on the wrong problem.
3. Procedural error: An error made during the execution of an appropriate and correctly identified procedure. The student understands what to do but makes mistakes (computational or algebraic) in doing it.
4. Technical error: An error attributable to carelessness or to a gap in foundational (prerequisite) mathematical knowledge — in arithmetic, basic algebra, or index laws rather than in complex number understanding per se.

These categories are not mutually exclusive in general, but in practice each identified error in this study was assigned a primary category based on its most proximal cause. The taxonomy provides both an analytical vocabulary and a diagnostic lens: differential patterns of error types across tasks reveal not only what students cannot do, but why.

Methodology

Research Design

This study employs a qualitative, interpretive research design. The primary data are students' handwritten responses to mathematical tasks. Frequency counts of error types are reported to provide an overview of distribution across tasks, but the core analytical work is interpretive: we examine specific student responses illustrated by figures to understand the reasoning processes, incomplete schemas, and procedural gaps that underlie each error type. This approach treats the student's written work as a text to be read, not merely a performance to be scored. It is acknowledged that all the 18 participants were drawn from a single course at a single institution and that the findings are therefore context-specific rather than statistically generalizable.

Participants

Participants were 18 students enrolled in MATH307 (Complex Analysis I), a compulsory course for third-year undergraduate mathematics education students at Ekiti State University, Ado-Ekiti, in affiliation with Emmanuel Alayande College of Education. All participants were full-time registered students pursuing a Bachelor of Science Education in Mathematics. This course is a prerequisite for more advanced complex analysis content; its successful completion is required for degree progression. All participants had come across complex numbers in one of their first-year undergraduate courses (Algebra and trigonometric). Participation in the study was voluntary. Ethical clearance for the study was obtained from the Research Ethics Committee of the Department of Mathematics, Emmanuel Alayande College of Education, Oyo. Participants were informed of the purpose of the study, assured of anonymity, and advised that their responses would be used solely for research purposes and would have no bearing on their course grades.

Instrument and Data Collection

Data were collected through a written task comprising six open-ended questions on introductory complex number topics. The instrument was designed to cover the full range of foundational complex number content in MATH307: algebraic simplification, the conjugate operation, division, multiplication, modulus and argument, and De Moivre's theorem. The actual questions, as administered to participants, are reproduced below.

1. Evaluate $4 - \frac{1}{2}i - 9 - \frac{5}{2}i$.
2. What is the conjugate of $12 + 7i$?
3. Simplify $\frac{3 + 2i}{1 - 4i}$.
4. Multiply $3 + 4i$ by $7 - 3i$.

5. Find the modulus and principal argument of $\frac{(1+i)^2}{1-i}$.

6. Express $\frac{(\cos\theta + i\sin\theta s^{8}}{(\sin\theta + i\cos\theta c^{8}}$ in the form $x + iy$.

Students were instructed to attempt all questions and show their working clearly. Their handwritten scripts were collected and retained as the primary data source.

At the time of data collection, students had completed the first three weeks of MATH307, during which instruction covered the definition and algebraic properties of complex numbers, the conjugate operation, modulus and argument, polar and exponential forms, and an introductory treatment of De Moivre's theorem. The written task was administered as a formative assessment at the end of this instructional block. The polar form conversion procedure had been taught most recently approximately one week before the assessment. This is relevant to interpreting the pattern of Interpretation errors observed across Questions 1, 2, and 3.

Data Analysis

Each student script was marked and all errors identified were assigned to one of the four error categories. Where a response contained multiple errors across different steps, each error was recorded and categorized independently. Blank responses were counted separately. Scripts in which all steps were correctly executed were categorized as 'No Error'. Frequency counts were compiled per question and across the full instrument. Representative student responses illustrating each error type were selected for qualitative analysis (typically one to two responses per error type per question) and are reproduced as figures in the findings section. The figures serve as the evidentiary basis for all interpretive claims made in the discussion. Selection prioritised responses that most clearly and unambiguously instantiated the error category, and where multiple students produced similar errors, the clearest example was chosen for illustration.

To establish reliability of the error categorisation, two members of the research team independently categorised the responses of six randomly selected students (one third of the total sample) using the four-category taxonomy. Initial agreement was calculated at 89%, and all discrepancies were resolved through discussion and consensus before final categorisation was applied to the full dataset. This process is consistent with qualitative inter-rater reliability procedures recommended for interpretive research of this kind (Siyepu, 2015).

Findings

Table 1 presents the distribution of error types across all six questions. A total of 108 individual responses were generated (18 students × 6 questions). Of these, 9 were left blank, 22 contained Conceptual errors, 17 contained Interpretation errors, 7 contained Procedural errors, 3 contained Technical errors, and 50 were correct. The following sections present a question-by-question analysis, moving from descriptive observation to interpretive analysis of the reasoning processes the errors reveal. Note that some responses contained errors of more than one type; in such cases the error was assigned to its primary category based on the most proximal cause, and the row totals for each question sum to 18.

Table 1

Distribution of Error Types Across Questions (N = 18)

Question	Blank	Conceptual	Interpretation	Procedural	Technical	No Error	Total
Q1	1	—	8	—	1	8	18
Q2	—	2	7	—	—	9	18
Q3	1	1	1	3	1	11	18
Q4	2	2	1	3	1	9	18
Q5	3	5	—	1	—	9	18
Q6	2	12	—	—	—	4	18
Total	**9**	**22**	**17**	**7**	**3**	**52**	**108**

To examine whether the distribution of response categories differed significantly across the six questions, a Pearson chi-square test of independence was conducted on the full 6 × 6 contingency table (questions × response categories, including Blank, Conceptual, Interpretation, Procedural, Technical, and No Error). The test yielded a statistically significant result, $\chi^2(25) = 69.37$, $p < .001$, confirming that the pattern of response categories was not uniform across questions i.e., different questions elicited qualitatively different profiles of errors and non-attempts. It should be noted that the chi-square approximation may be unreliable in this context due to several cells with expected frequencies below 5, a consequence of the small sample size (N = 18). The result should therefore be interpreted with caution and treated as indicative rather than definitive. Notwithstanding this caveat, the finding is consistent with and supportive of the qualitative patterns identified in the question-by-question analysis: specifically, that Interpretation errors were concentrated in Questions 1–3, Conceptual errors were disproportionately concentrated in Question 6, and the blank response rate increased with question difficulty. The substantive interpretation of these patterns rests on the qualitative analysis of student work presented in the sections that follow, for which the statistical result provides corroborating support.

Question 1

Evaluate $4 - \frac{1}{2}i - 9 - \frac{5}{2}i$

Blank	Conceptual	Interpretation	Procedural	Technical	No Error
1	—	8	—	1	8

This task required students to collect and simplify real and imaginary parts separately. The correct solution proceeds as follows: the real parts give $4 - 9 = -5$, and the imaginary parts give $-\frac{1}{2} - 5/2 = -(1/2 + 5/2) = -6/2 = -3$, yielding $z = -5 - 3i$. This is a straightforward algebraic grouping task. Eight students answered correctly; nine errors were recorded — eight Interpretation and one Technical with one response left blank.

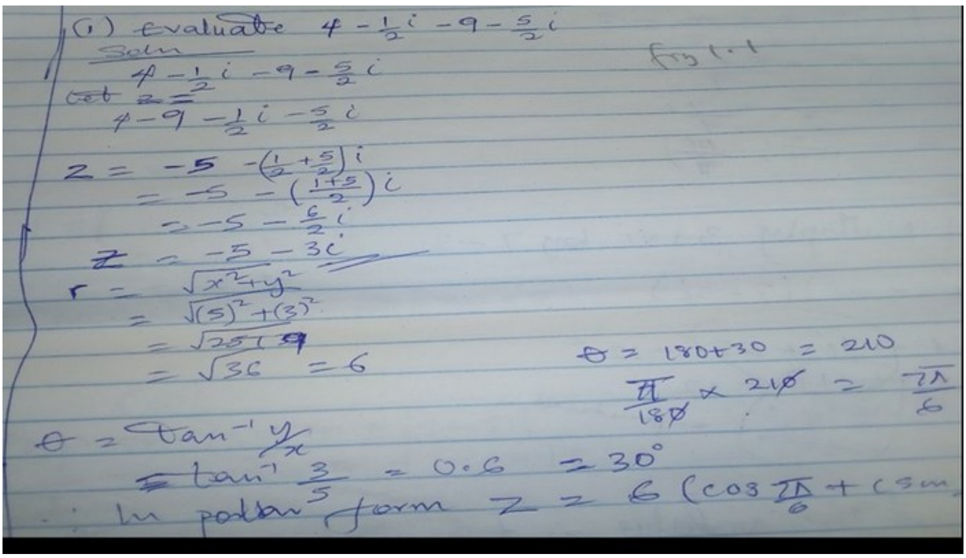
The complete absence of Conceptual and Procedural errors might indicate that students at this level have sufficient structural understanding of complex number form ($a + bi$) to group real and imaginary parts. The difficulty is not conceptual but operational specifically, in task discrimination and in basic arithmetic.

Interpretation Error (n = 8)

Figure 1.1 illustrates the dominant error pattern. The student correctly performs the grouping, arriving at z = −5 − 3i, and then immediately proceeds to compute the modulus and argument: $r = \sqrt{5^2 + 3^2}, \theta = \tan^{-1}\frac{3}{5}$ adjusted to the third quadrant, giving the final answer in polar form $z = 6(\cos\frac{7\pi}{6} + isin\frac{7\pi}{6})$. Two observations are critical here. First, the procedure applied i.e., finding modulus and argument is itself mathematically competent; the student executes it with reasonable fluency. Second, and more importantly, the student has not read what the task is asking. The instruction is to evaluate (simplify) the expression, not to convert it to polar form. The correct answer, z = −5 − 3i, is actually visible in the student's working and then discarded in favour of the unnecessary continuation.

Figure 1.1

Interpretation error in Question 1: Student correctly simplifies to z = −5 − 3i, then redundantly computes modulus and principal argument, expressing the final answer in polar form rather than stopping at the simplified expression.



It is also worth noting that the student's modulus computation contains an arithmetic error: $\sqrt{25 + 9} = \sqrt{36}$ is rounded or miscomputed as $\sqrt{36} = 6$. This embedded arithmetic slip within an interpretation error illustrates how errors of different types can co-occur within a single response. The dominant classification here is Interpretation (the student's principal failure is not knowing when to stop) but the embedded arithmetic error would, in isolation, constitute a Technical error.

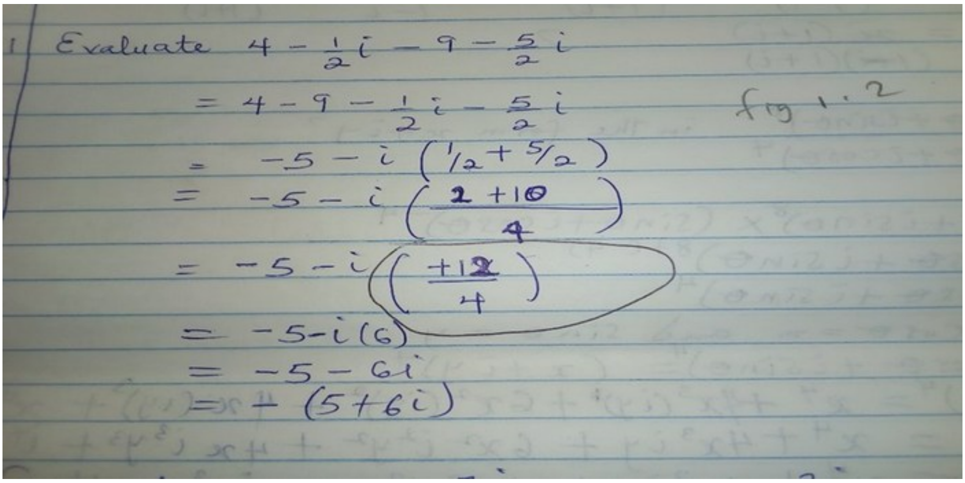
This pattern of correct simplification followed by unnecessary polar conversion appeared in eight of eighteen responses. Its consistency across nearly half the class strongly suggests that at the time of assessment, polar form conversion was the most recently and most prominently taught procedure, and that students defaulted to it regardless of task demands. This is a textbook manifestation of what Olivier (2003) terms proactive interference: a more salient or recently acquired schema suppresses the simpler, contextually appropriate response.

Technical Error (n = 1)

Figure 1.2 illustrates the Technical error. The student correctly identifies that the imaginary parts must be combined ($\frac{1}{2}+\frac{5}{2}$) and sets up the fraction addition as ½ + $^5/_2$ = (2 + 10)/4 =$^{12}/_4$. This is a conversion that is itself structurally correct. However, the student then incorrectly simplifies $^{12}/_4$ as 6, dividing 12 by 2 rather than by 4. This is a straightforward arithmetic slip in the final simplification step: 12 ÷ 4 = 3, not 6. The conceptual structure of the complex simplification is entirely intact i.e., the student correctly identifies which parts to combine and correctly finds the common denominator but the failure to divide accurately at the last step yields the wrong imaginary coefficient. The student arrives at z = −5 − 6i instead of the correct −5 − 3i.

Figure 1.2

Technical error in Question 1: Correct grouping structure applied, but the student simplifies 12/4 as 6 rather than 3 — dividing by 2 instead of 4 — yielding the wrong imaginary coefficient and a final answer of z = −5 − 6i.



Question 2

What is the Conjugate of 12 + 7i?

Blank	Conceptual	Interpretation	Procedural	Technical	No Error
—	2	7	—	—	9

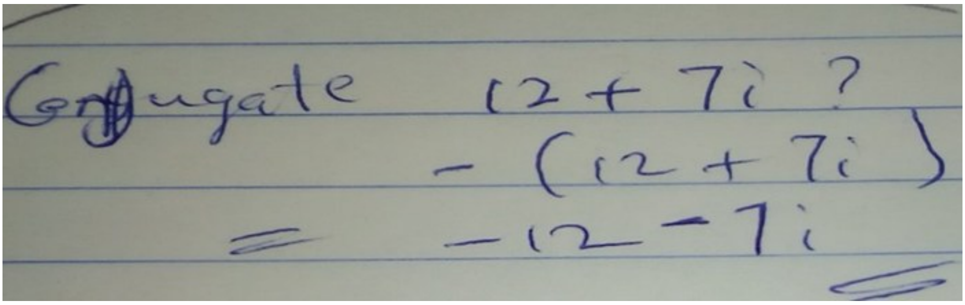
This task required students to state the complex conjugate of 12 + 7i. The answer is simply 12 − 7i: the conjugate of z = a + bi is z̄ = a − bi, in which only the sign of the imaginary part changes. Nine students answered correctly. Nine errors were recorded: two Conceptual and seven Interpretation. The absence of Procedural and Technical errors is telling once a student correctly understands what the conjugate operation is, there is essentially no calculation to make an error in. The errors observed here are therefore entirely about conceptual misunderstanding or task misidentification.

Conceptual Error (n = 2)

Figure 2.1 shows a student who wrote the conjugate of 12 + 7i as −(12 + 7i) = −12 − 7i. This student has negated both the real and imaginary parts confusing the conjugate with the additive inverse of the complex number. The error reveals an incomplete and incorrect concept image of the conjugate operation. The student knows that 'something changes sign' when finding the conjugate, but has not correctly internalized that it is exclusively the imaginary part that changes. This is not a reading error or a procedural slip; it reflects a genuine misconception about what the conjugate means.

Figure 2.1

Conceptual error in Question 2: Student writes the conjugate of 12 + 7i as −12 − 7i, negating both real and imaginary parts rather than only the imaginary part — confusing the conjugate operation with the additive inverse.



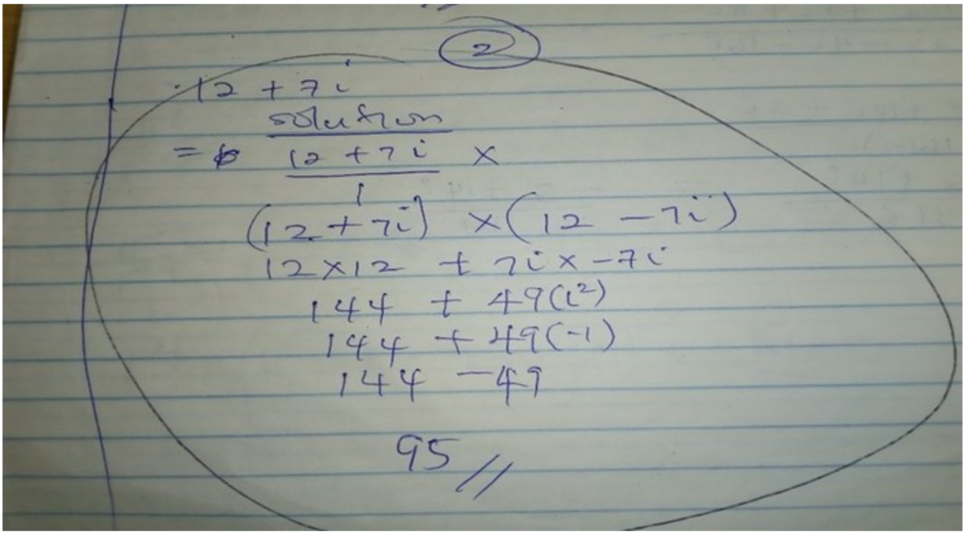
Interpretation Errors (n = 7)

The seven Interpretation errors fell into two distinct sub-types, both of which share the same root cause: the student identifies the conjugate (12 − 7i) correctly as part of a related procedure, but then proceeds to complete that procedure rather than stopping at the answer.

The first and more prevalent sub-type is illustrated in Figure 2.2. The student computes $(12 + 7i)(12 - 7i) = 144 + 12(-7i) + 7i(12) + 49i^2 = 144 - 49 = 95$. This is the product of a complex number and its conjugate, a procedure that yields the modulus squared ($|z|^2 = a^2 + b^2 = 144 + 49 = 193$; the student's answer of 95 = 144 − 49 contains a further arithmetic error, treating $i^2 = -1$ only partially). What is analytically important is not the arithmetic but the procedural choice: the student has activated the procedure for rationalization (multiplying by the conjugate) rather than the simpler operation of merely stating the conjugate. This is the same overgeneralization pattern seen in Question 1 — a more complex, recently prominent procedure displaces the simpler appropriate one.

Figure 2.2

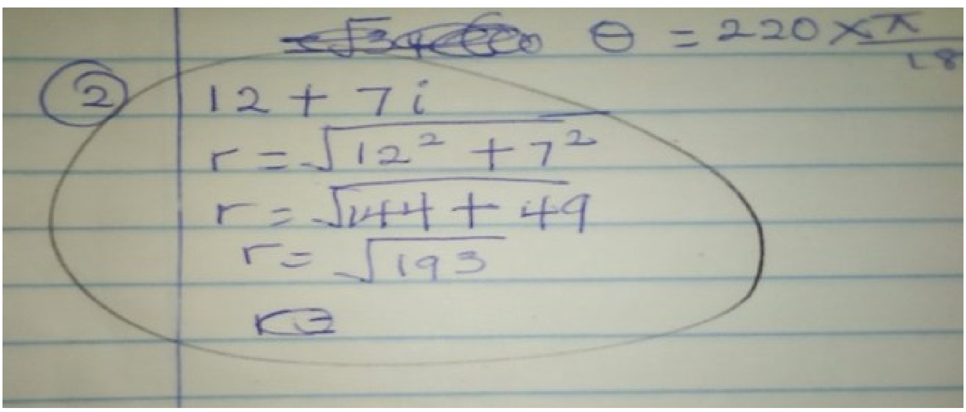
Interpretation error in Question 2: Student correctly identifies 12 − 7i as the conjugate but proceeds to compute z · z̄ = (12 + 7i)(12 − 7i) = 95, substituting the rationalization procedure for the simpler task of merely stating the conjugate.



The second sub-type is shown in Figure 2.3. Here, the student computes $r = \sqrt{12^2 + 7^2} = \sqrt{144 + 49} = \sqrt{193}$, the modulus of the complex number. Again, the student has activated a related but contextually inappropriate procedure (modulus calculation) instead of the conjugate. The pattern across both sub-types confirms that students at this level tend to read a complex number expression and search for a procedure that 'does something' to it, rather than discriminating carefully between conjugate, modulus, and product operations.

Figure 2.3.

Interpretation error in Question 2: Student computes the modulus r = $\sqrt{193}$ *rather than stating the conjugate — a second sub-type of interpretation error in which the modulus procedure is substituted for the conjugate operation.*



The combined pattern across Questions 1 and 2 interpretation errors involving substitution of modulus-argument or polar-form procedures strongly suggests that these procedures had been taught most recently and most prominently, and had become the default response to any complex number task. This finding aligns with Karakok et al.'s (2015) argument that introducing complex number procedures in rapid succession, without ensuring that students can discriminate between them, creates conditions for systematic overgeneralization.

Question 3

Simplify $\frac{3+2i}{1-4i}$

Blank	Conceptual	Interpretation	Procedural	Technical	No Error
1	1	1	3	1	11

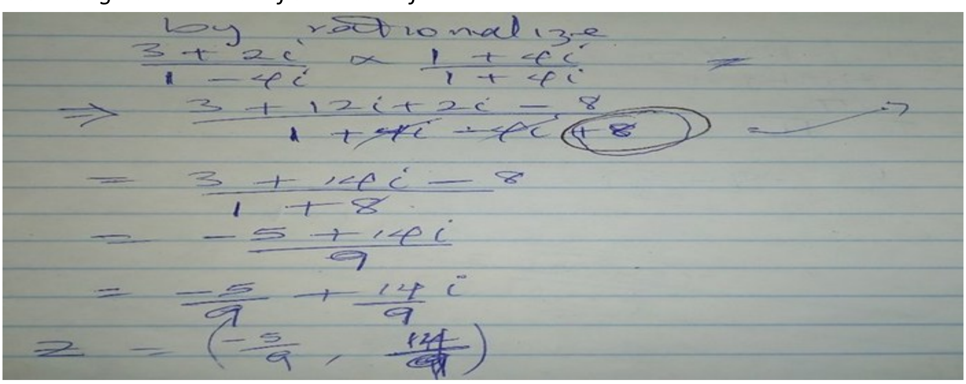
Division of complex numbers requires rationalization: multiplying numerator and denominator by the conjugate of the denominator. For $\frac{3+2i}{1-4i}$, the conjugate of the denominator is (1 + 4i). The correct solution proceeds as $\frac{3+2i\,(1+4i)}{1-4i(1+4i)} = \frac{3+12i+2i+8i^2}{1+16} = \frac{-5+14i}{17} = -\frac{5}{17} + \frac{14}{17}i$. Eleven students answered correctly; six errors were distributed across all four categories; one response was blank.

Technical Error (n = 1)

Figure 3.1 illustrates the Technical error. The student correctly identifies the conjugate of (1 − 4i) as (1 + 4i) and sets up the rationalization correctly. In computing the denominator (1 − 4i)(1 + 4i), however, the student evaluates $(4i)^2$ as 4 + 4 = 8 rather than 16 (since $(4i)^2 = 16i^2 = -16$, making the denominator 1 − (−16) = 17). The student's denominator becomes 1 + 8 = 9, yielding the wrong final answer of $\frac{-5+14i}{9}$. The setup is entirely correct; the rationalization strategy is clearly understood. The error is a specific arithmetic failure in squaring 4i, the student appears to compute 4 + 4 instead of 4 × 4, a careless conflation of addition and multiplication.

Figure 3.1

Technical error in Question 3: Student correctly identifies (1 + 4i) as the conjugate of (1 − 4i) and sets up the rationalization, but computes $(4i)^2$ as 4 + 4 = 8 rather than 16, yielding the wrong denominator of 9 instead of 17.

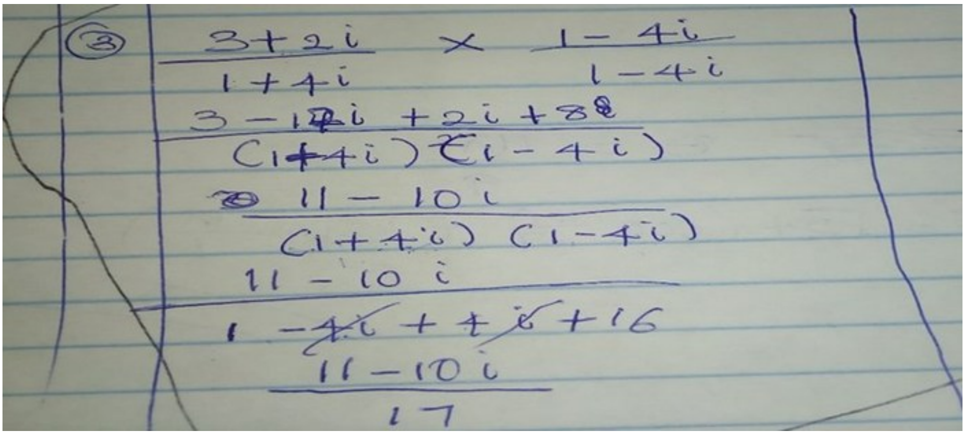


Procedural Errors (n = 3)

Figure 3.2 illustrates a representative Procedural error. The student multiplies (3 + 2i) by the conjugate (1 − 4i) but uses the wrong sign: the conjugate of (1 − 4i) is (1 + 4i), not (1 − 4i). This means the student is multiplying both numerator and denominator by (1 − 4i), which does not rationalize the denominator. The expansion of the numerator proceeds: $(3 + 2i)(1 - 4i) = 3 - 12i + 2i - 8i^2 = 3 + 8 - 10i = 11 - 10i$, which the student correctly computes. The denominator (1 + 4i)(1 − 4i), note that the student appears to use the correct denominator product in the denominator line while using the wrong conjugate in the numerator yields 1 + 16 = 17. The error here is procedural: the student understands rationalization conceptually but incorrectly identifies the conjugate to multiply by, a failure in executing the first step of the procedure.

Figure 3.2

Procedural error in Question 3: Student multiplies by (1 − 4i)/(1 − 4i) instead of (1 + 4i)/(1 + 4i), choosing the wrong conjugate for rationalization (understanding the strategy but incorrectly executing the first step).



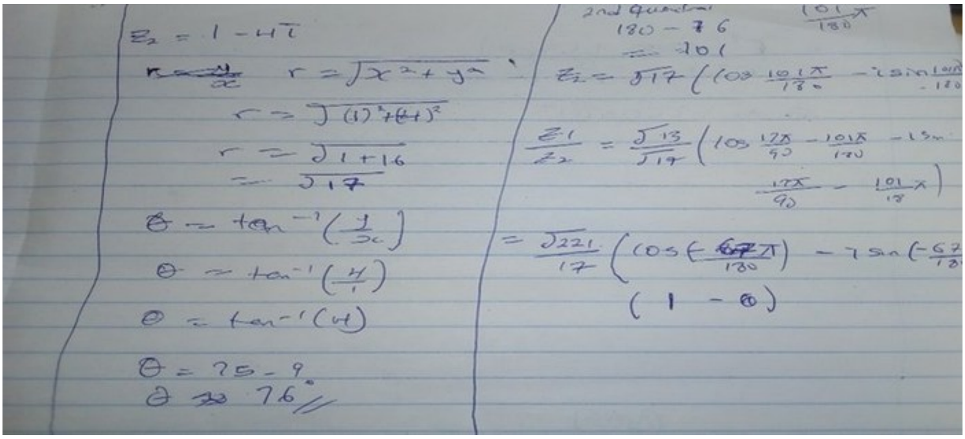
Interpretation Error (n = 1)

Figures 3.3a and 3.3b illustrate the Interpretation error. Rather than simplifying the division algebraically into a + bi form, the student converts both (3 + 2i) and (1 − 4i) to polar form separately computing modulus and argument for each and then divides them in polar form, expressing the result as $\frac{r_1}{r_2}[(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$. While this approach is mathematically legitimate, it is contextually inappropriate: the task asks to simplify, and the expected form is a + bi. More importantly, this is the same pattern activating a polar-form procedure in response to a task that does not require it — that appeared in Questions 1 and 2. By Question 3, this pattern has been observed in 16 instances across three questions, and it appears across different students, confirming that it is a class-wide tendency rather than an idiosyncratic habit.

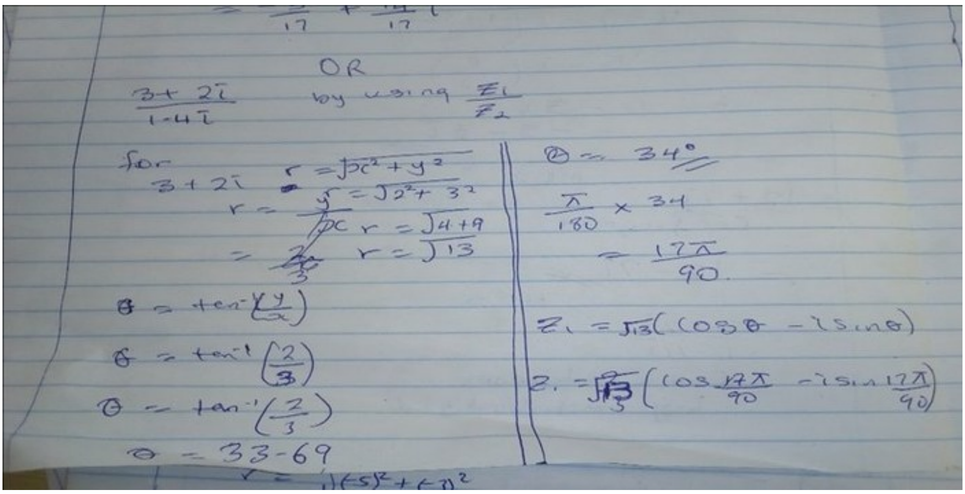
Figures 3.3a & 3.3b.

Interpretation error in Question 3: Student converts both complex numbers to polar form separately and divides them in polar form, rather than rationalizing the expression to obtain the result in standard a + bi form.

a



b



The distribution of errors across all four categories in this question despite its being the task with the highest correct rate (11/18) confirms that complex number division is simultaneously the most tractable and the most procedurally vulnerable task type in the instrument. Most students who know the rationalization procedure execute it correctly; most errors arise from not knowing it (Conceptual), misidentifying the conjugate (Procedural), or not recognizing that it is needed (Interpretation).

Question 4

Multiply 3 + 4i by 7 − 3i

Blank	Conceptual	Interpretation	Procedural	Technical	No Error
2	2	1	3	1	9

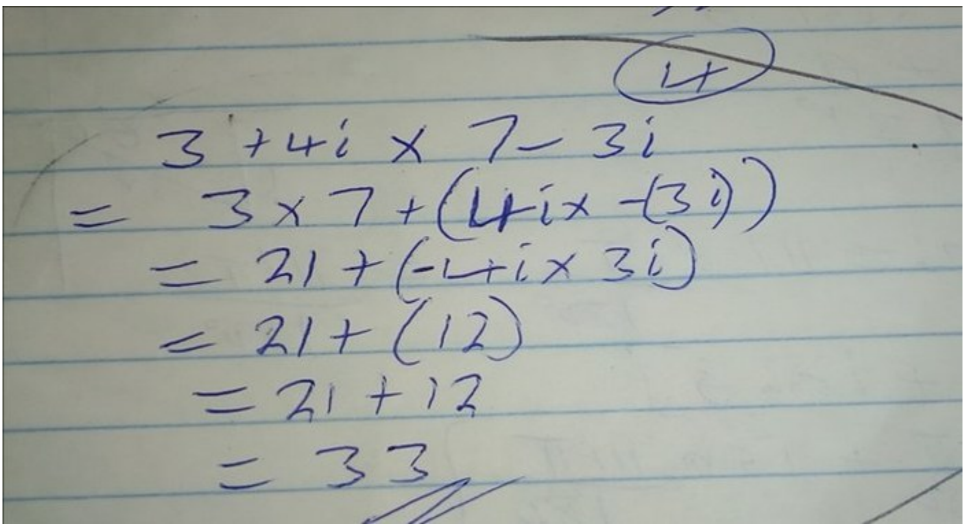
Multiplication of complex numbers requires full application of the distributive property (FOIL) and substitution of $i^2 = -1$. The correct solution is: $(3 + 4i)(7 - 3i) = 3(7) + 3(-3i) + 4i(7) + 4i(-3i) = 21 - 9i + 28i - 12i^2 = 21 + 12 + (-9 + 28)i = 33 + 19i$. Nine students answered correctly. Seven errors were recorded: one Technical, one Interpretation, three Procedural, and two Conceptual. Two responses were blank.

Technical Error (n = 1)

Figure 4.1 shows the Technical error. The student writes $(3 + 4i)(7 - 3i) = 3 \times 7 + (4i \times -3i) = 21 + 12 = 33$. Two things are happening simultaneously. First, the student omits the two cross-terms entirely ($3 \times -3i = -9i$ and $4i \times 7 = 28i$ are both missing), suggesting a failure to apply the full distributive property. Second, the student computes $4i \times -3i = 12$ which is numerically correct (since $4 \times -3 \times i^2 = -12 \times -1 = 12$) but treats this as a real number result without recognizing that it involves $i^2 = -1$. The student appears to have applied a 'shortcut' that conflates the real and imaginary multiplications, arriving at 33 with no imaginary part. This combination of missing cross-terms and opaque handling of i^2 is best classified as a Technical error rooted in careless application of the distributive property.

Figure 4.1

Technical error in Question 4: Student computes only 3 × 7 = 21 and 4i × (−3i) = 12, omitting both cross-terms (−9i and 28i) entirely from the distributive expansion, yielding the real-valued but incomplete answer of 33.



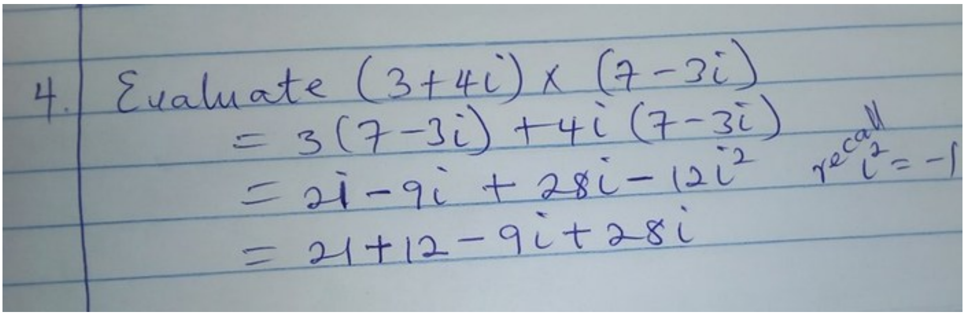
Correct Solution (for contrast)

Figures 4.2a and 4.2b illustrate a fully correct solution. The student correctly expands all four terms of (3 + 4i)(7 − 3i), applies $i^2 = -1$ explicitly (annotating 'recall $i^2 = -1$'), combines like terms, and arrives at 33 + 19i. This response is included here as an analytical contrast: it demonstrates that the correct procedure is accessible to students at this level and that the errors observed in other responses are not inevitable given the cognitive demands of the task.

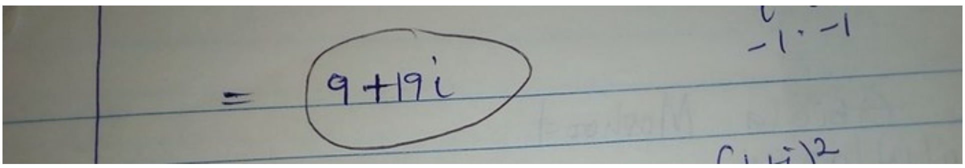
Figures 4.2a & 4.2b

Correct solution to Question 4: Student applies full distributive expansion, explicitly notes $i^2 = -1$, correctly combines all four terms, and arrives at the correct answer 33 + 19i — included as an analytical contrast to the erroneous responses.

a



b

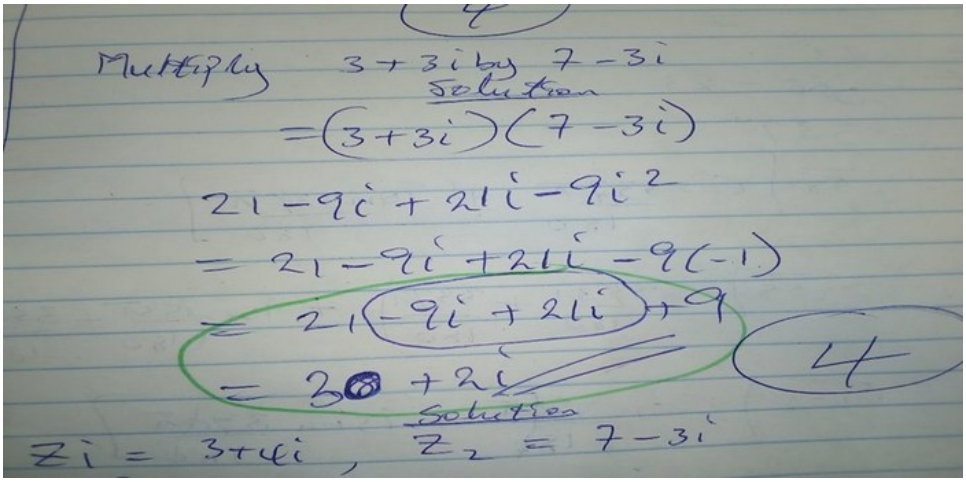


Procedural Errors (n = 3)

Figure 4.3 illustrates a representative Procedural error. The student sets up the expansion correctly and applies $i^2 = -1$, but makes an arithmetic error in combining the imaginary terms: −9i + 21i is written as 2i rather than 12i (note: this student appears to have worked with 3 + 3i rather than 3 + 4i in the original expression, which itself may reflect a copying error or a misreading of the question). The structural understanding of the multiplication procedure is intact; the failure is in arithmetic simplification. The conceptual scaffolding i.e., knowing to expand fully, knowing to substitute $i^2 = -1$ is present; the execution breaks down at the final collection of like terms.

Figure 4.3

Procedural error in Question 4: Student correctly expands all terms and applies $i^2 = -1$, but combines the imaginary terms incorrectly (writing −9i + 21i as 2i rather than 12i — yielding the wrong final answer).



Conceptual Errors (n = 2)

The two Conceptual errors (not separately illustrated) revealed a failure to understand the behaviour of integer powers of i. One student expressed i^2 as a value other than −1 in the course of the expansion, demonstrating that the foundational identity $i^2 = -1$ had not been consolidated. A second student incorrectly applied index laws to imaginary expressions, treating i as if it obeyed real-number index rules without regard to the cyclical pattern ($i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$). These are not arithmetic slips: they reflect conceptual gaps in understanding what i is and how it behaves algebraically. Such gaps will propagate into any complex number task involving higher powers — most acutely into Question 6.

Question 5: Find the Modulus and Principal Argument of $\frac{(1+i)^2}{1-i}$.

Blank	Conceptual	Interpretation	Procedural	Technical	No Error
3	5	—	1	—	9

This question is the most structurally layered task in the instrument. It requires students to: (i) first simplify the complex quotient $\frac{(1+i)^2}{1-i}$ into standard a + bi form which requires expanding the numerator and rationalizing the denominator; (ii) then apply the modulus formula $|z| = \sqrt{a^2 + b^2}$; and (iii) then compute the principal argument $\theta = \tan^{-1}\frac{b}{a}$ adjusted for the correct quadrant. The complete correct solution is: $(1 + i)^2 = 1 + 2i + i^2 = 2i$; then $2i/(1 - i) \times (1 + i)/(1 + i) = (2i + 2i^2)/(1 + 1) = (2i - 2)/2 = i - 1 = -1 + i$. Hence modulus = $\sqrt{1 + 1} = \sqrt{2}$, and principal argument = $\pi - \pi/4 = 3\pi/4$ (second quadrant). Nine students answered correctly. Five Conceptual errors and one Procedural error were recorded; three responses were blank.

The absence of Interpretation errors is noteworthy and contrasts with Questions 1–3. For this task, students correctly identified what was being asked (modulus and argument). The difficulty lay not in task identification but in the prerequisite computation, specifically, simplifying the quotient.

Conceptual Errors (n = 5)

Figure 5.1 illustrates the most severe Conceptual error pattern. The student begins to simplify $\frac{(1+i)^2}{1-i}$ but quickly abandons the algebraic approach. Instead, he rewrites the expression as $(\cos\theta + i\sin\theta)^8/(\sin\theta + i\cos\theta)^4$ spontaneously substituting expressions from Question 6's form into Question 5. This response reveals two simultaneous conceptual failures: the student cannot simplify the given quotient, and he conflates the task structure of Question 5 with that of Question 6. The confusion between questions suggests that De Moivre's theorem learned as a formula for expressions of the form $(\cos\theta + i\sin\theta)^n$ has become a kind of cognitive magnet that attracts any complex expression the student cannot simplify by other means.

Figure 5.1

Conceptual error in Question 5: Student cannot simplify the complex quotient $(1 + i)^2/(1 - i)$ and instead substitutes expressions from Question 6's form — writing $(\cos\vartheta + i\sin\vartheta)^8/(\sin\vartheta + i\cos\vartheta)^4$ — conflating two entirely different tasks and demonstrating deep procedural confusion.

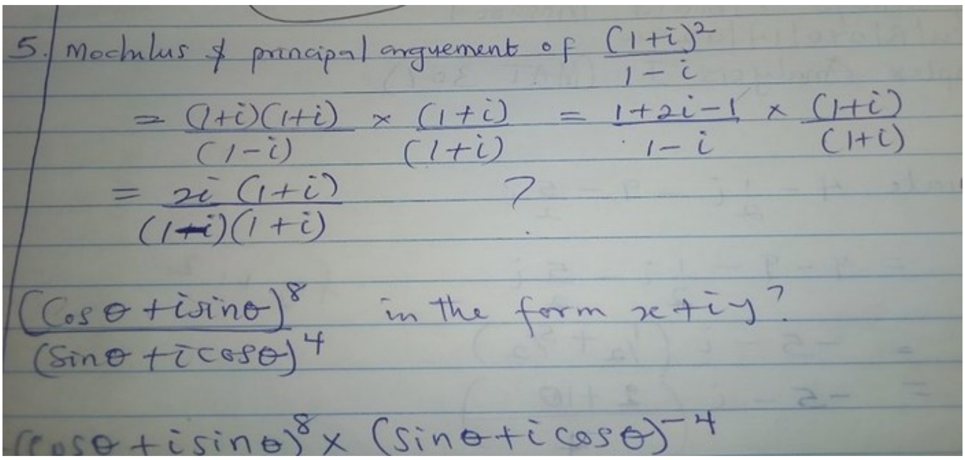
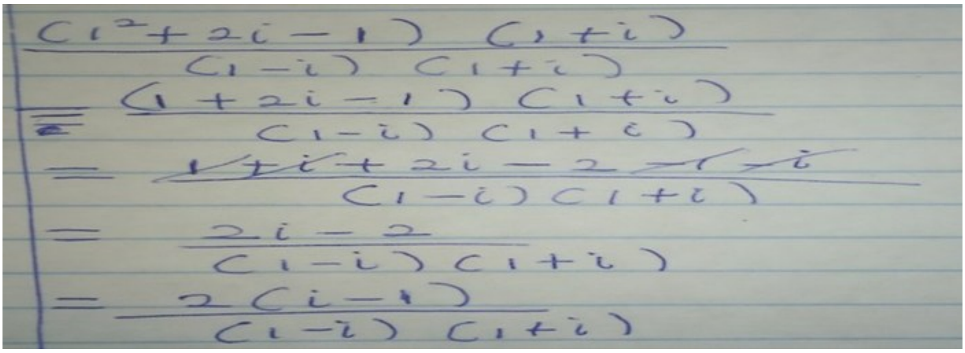


Figure 5.2 illustrates a different but related Conceptual error. The student correctly expands $(1 + i)^2 = (1 + i)(1 + i)$ and simplifies the numerator: $1 + i + i + i^2 = 1 + 2i - 1 = 2i$. The student then writes the expression as $\frac{2i}{1-i}$ and rationalizes: multiplying by $\frac{1+i}{1-i}$ gives $(2i + 2i^2)/((1)^2 + (1)^2) = (2i - 2)/2 = i - 1 = -1 + i$. This simplification is actually correct. However, the student then attempts to find the modulus by writing $r = \sqrt{(2i^2 + (1 - i)^2)}$ substituting the pre-simplified form of the expression into the modulus formula as if $2i$ and $(1 - i)$ were the real and imaginary coordinate values. The student has done the

hard work correctly but then applies the modulus formula to the wrong object. This reveals a specific conceptual gap: the student understands that modulus requires $\sqrt{a^2 + b^2}$, but has not grasped that a and b must be the real and imaginary parts of the simplified a + bi form, not the components of the unsimplified quotient.

Figure 5.2

Conceptual error in Question 5: Student correctly expands the numerator to obtain 2i but cannot complete the rationalization of the denominator, stalling before the modulus and argument can be computed — revealing that failure at complex division (Question 3) has propagated forward.

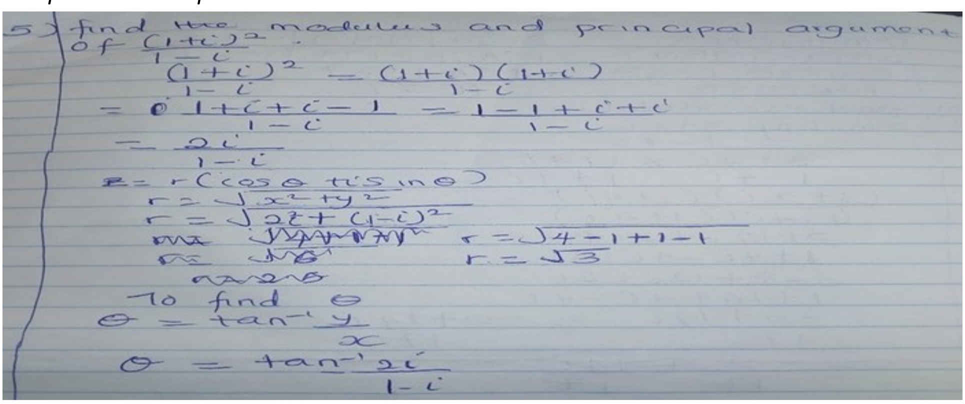


Procedural Error (n = 1)

Figure 5.3 illustrates the Procedural error. The student correctly expands $(1 + i)^2 = 2i$ and sets up the rationalization $2i(1 + i)/((1 - i)(1 + i)) = (2i + 2i^2)/2 = (2i - 2)/2 = i - 1 = -1 + i$. The simplification is correct. The student then correctly identifies that modulus $= \sqrt{a^2 + b^2}$ and argument $= \tan^{-1}\frac{b}{a}$. However, in substituting values, he makes a computational error using incorrect values for a and b, likely from an intermediate step and arrive at a wrong numerical answer for both modulus and argument. The conceptual pathway is intact; the failure is in the final numerical substitution.

Figure 5.3

Procedural error in Question 5: Student correctly simplifies the quotient to −1 + i but then substitutes incorrect values into the modulus and argument formulae in the final computational step.



The results from Question 5 carry an important diagnostic message: the three blank responses and five Conceptual errors are largely traceable to the failure of complex division as a prerequisite skill. Students who could not rationalize in Question 3 could not set up Question 5 at all. The cumulative dependency between these two questions is not incidental — it reflects the inherent hierarchical structure of complex number knowledge, in which division is foundational to a range of subsequent operations.

Question 6: Express $\frac{(\cos\theta + i\sin\theta s^{8}}{(\sin\theta + i\cos\theta c^{8}}$ **in the form x + iy**

Blank	Conceptual	Interpretation	Procedural	Technical	No Error
2	12	—	—	—	4

This task required students to simplify a quotient involving trigonometric complex expressions and express the result in standard x + iy form. The key insight is to rewrite the denominator: sinθ + i cosθ = cos(π/2 − θ) + i sin(π/2 − θ) = e^{i(π/2−θ)}, or equivalently, sinθ + i cosθ = i(cosθ − i sinθ) + ... More directly: note that sinθ + i cosθ = i(cosθ − i sinθ) is not immediately useful; the cleaner approach recognizes that sinθ + i cosθ = cos(π/2 − θ) + i sin(π/2 − θ), so by De Moivre's theorem $(\sin\theta + i\cos\theta)^8 = (\cos(\pi/2 - \theta) + i\sin(\pi/2 - \theta))^8$ = cos(8(π/2 − θ)) + i sin(8(π/2−θ)) = cos(4π − 8θ) + i sin(4π − 8θ) = cos(−8θ) + i sin(−8θ) since cos(4π − 8θ) = cos(8θ) and sin(4π − 8θ) = −sin(8θ). The numerator $(\cos\theta + i\sin\theta)^8$ = cos(8θ) + i sin(8θ). The quotient is then [cos8θ + i sin8θ]/[cos8θ − i sin8θ] = cos(16θ) + i sin(16θ) after division using De Moivre giving x + iy = cos16θ + i sin16θ. This task produced the most striking finding: 1 of 18 students committed Conceptual errors, and only 4 answered correctly.

The analytical significance of this result lies not only in its magnitude but in its uniformity. No Interpretation, Procedural, or Technical errors were recorded. Students either had conceptual access to De Moivre's theorem and the necessary polar form reasoning, or they did not. The binary character of the distribution — four students with full conceptual access, twelve with none — is qualitatively different from the error distributions in Questions 1–5, where errors were distributed across multiple types and most students achieved at least partial progress.

Conceptual Errors (n = 12)

The 12 Conceptual errors were largely uniform in character: students could not initiate the solution process. Unlike Questions 3 and 5, where students at least attempted the first step before making an error, most students here either left the response blank (n = 2 of the 12) or wrote attempts that bore no structural relationship to the relevant procedure. The expression $(\cos\theta + i\sin\theta)^8/(\sin\theta + i\cos\theta)^8$ appeared to be unrecognizable as a De Moivre's theorem problem to most of the cohort.

This finding is consistent with Danenhower's (2006) APOS-based conclusion that students who have action-level competence with individual complex number operations i.e., that is, students who can compute modulus, argument, and rectangular form separately separately frequently lack the schema-level understanding needed to coordinate these operations or apply them in novel configurations. De Moivre's theorem requires students to see an expression like $(\cos\theta + i\sin\theta)^n$ as an encoding of a complex number in polar form and to recognize that raising it to a power corresponds to a specific operation on that encoding. Most students in this cohort had not reached this level of integration.

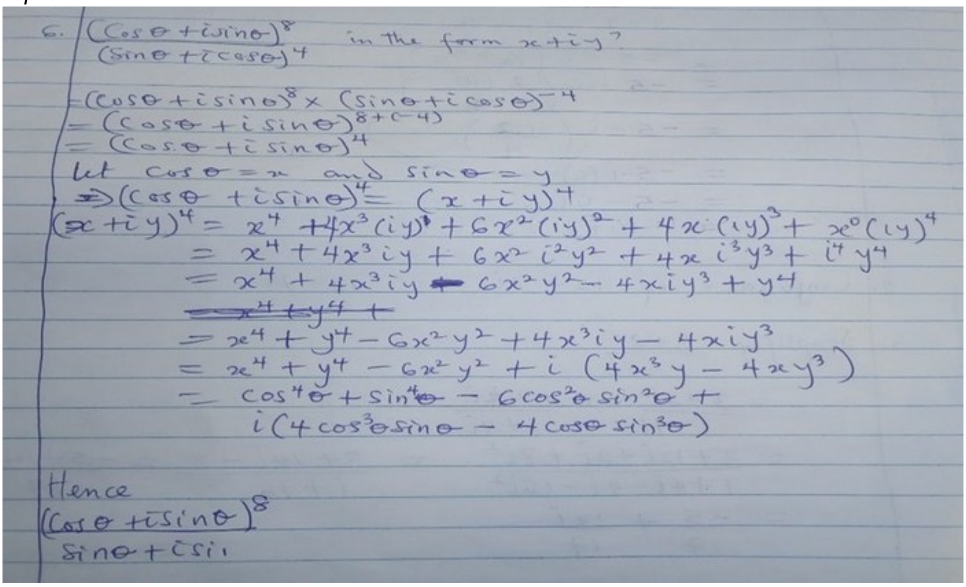
Correct Solutions (n = 4)

Figure 6.1 illustrates one of the four correct responses. The student correctly identifies that the denominator can be rewritten in a compatible polar form, reduces the quotient using index laws for complex exponentials, and then expands $(\cos\theta + i \sin\theta)^4$ using the binomial theorem — substituting $x = \cos\theta$ and $y = \sin\theta$ and carefully tracking powers of i ($i^2 = -1$, $i^3 = -i$, $i^4 = 1$) through the expansion. The final result is expressed in terms of powers of cosθ and sinθ, correctly separated into real and imaginary parts. This response demonstrates not just knowledge of De Moivre's theorem as a formula but genuine mathematical fluency in working across representations — exactly the kind of coordinated understanding that Danenhower (2006) and Nordlander and Nordlander (2012) identify as the hallmark of mature complex number competence.

It is worth considering what distinguished these four students from their twelve peers who could not initiate the solution. While the study design does not permit definitive causal claims i.e., no follow-up interviews were conducted with the successful students, three tentative observations can be drawn from their written work. First, all four correct responses show evidence of systematic, step-by-step working: students wrote out every intermediate step explicitly rather than attempting shortcuts, suggesting stronger metacognitive monitoring of their own solution process. Second, all four responses correctly identified the key first move which is recognising that $\sin\theta + i\cos\theta = \cos(\pi/2 - \theta) + i\sin(\pi/2 - \theta)$ and rewriting the denominator in compatible polar form before proceeding. This indicates that these students had internalised De Moivre's theorem not as an isolated formula but as part of a broader understanding of polar representation and the relationships between trigonometric functions. Third, the binomial expansion in all four correct responses was carried out with careful and explicit tracking of the powers of i ($i^2 = -1$, $i^3 = -i$, $i^4 = 1$) at every step, suggesting that these students had also consolidated the foundational identity $i^2 = -1$ and its cyclic consequences more thoroughly than their peers. These observations suggest that success on Question 6 was not attributable to any single factor but to the convergence of several competencies: secure polar form understanding, consolidated knowledge of the powers of i, and the metacognitive discipline to work systematically through a multi-step problem.

Figure 6.1

Correct solution to Question 6: Student identifies that the denominator sin ϑ + i cos ϑ = cos(π/2 − ϑ) + i sin(π/2 − ϑ), applies De Moivre's theorem to both numerator and denominator, simplifies the quotient, and expands using the binomial theorem with careful tracking of powers of i demonstrating schema-level understanding of polar representation.



The contrast between the four correct responses and the twelve errors in Question 6 encapsulates the central challenge of complex number instruction at undergraduate level: De Moivre's theorem is not simply a formula to be memorized and applied, but the culmination of a conceptual journey through polar representation, the geometry of complex multiplication, and the coordination of multiple forms. Students who have not made that journey cannot apply the theorem and, as the blank responses suggest, many are aware that they cannot.

Cross-Cutting Patterns and Theoretical Implications

Three overarching patterns emerge from reading across all six questions, each carrying theoretical and pedagogical significance.

First, the systematic recurrence of Interpretation errors involving polar-form or modulus-argument substitution across Questions 1, 2, and 3 appearing in 16 of 54 responses to these three questions constitutes strong evidence of schema-driven overgeneralization. Students have acquired the modulus-argument and polar-form procedures as salient, prominent schemas and apply them as default responses to any complex number task, regardless of what the task specifies. This is not a random error; it is a systematic cognitive bias of the kind described by Olivier (2003) and predictable from schema theory. The pedagogical implication is that instruction must actively address task discrimination not just teach procedures, but explicitly teach students to read tasks carefully and distinguish between what different tasks require.

Second, the data reveal a clear cumulative dependency structure that mirrors the hierarchical organization of complex number knowledge. The rationalization procedure (Question 3) is a prerequisite for Question 5; the identity $i^2 = -1$ and its consequences (Question 4) are prerequisites for Question 6. Errors in these foundational tasks do not stay contained, they propagate forward. This has a direct implication for formative assessment: instructors need diagnostic information about students' mastery of prerequisite skills before introducing dependent topics.

Third, the binary distribution of performance on Question 6 i.e., four fully correct, twelve unable to begin, with no partial-credit responses in between suggests that De Moivre's theorem operates as a threshold concept in the sense of Meyer and Land (2003): a conceptual gateway that, once crossed, transforms students' understanding, but before which progress is blocked. The absence of any Procedural or Technical errors in Question 6 means that students who lacked conceptual access could not even generate incorrect procedural attempts. This is qualitatively different from the error profiles of Questions 1–5 and implies that the instructional approach to De Moivre's theorem needs to be fundamentally different from the approach to other complex number operations less formula-focused and more concept-building.

Discussions

The findings of this study extend and deepen the existing literature on students' difficulties with complex numbers in several important ways.

First, the systematic recurrence of Interpretation errors across Questions 1, 2, and 3 in which students substituted modulus-argument and polar-form procedures for simpler operations mirrors overgeneralization patterns documented in other areas of undergraduate mathematics. In calculus, Orton (1983) observed that students routinely applied differentiation rules beyond their valid domain; in integration, Kiat (2005) found that students defaulted to familiar integration techniques regardless of whether they were appropriate. The present study suggests that this schema-driven overgeneralization is not domain-specific but reflects a broader cognitive tendency that emerges whenever students acquire a salient new procedure before fully consolidating task-discrimination skills. This has implications beyond complex numbers: it suggests that any undergraduate mathematics course introducing multiple related procedures in rapid succession is at risk of producing similar Interpretation error patterns.

Second, the cumulative dependency structure observed here whereby failure at complex division (Question 3) propagated into failure at the modulus-argument task (Question 5) is consistent with findings in linear algebra education, where researchers have similarly documented how gaps in foundational matrix operations impair students' ability to engage with eigenvalue problems (Seloane et al., 2023). This structural parallel suggests that the hierarchical organization of mathematical knowledge is a general feature of undergraduate mathematics learning, not a peculiarity of complex number topics, and that formative diagnostic assessment of prerequisite skills is a broadly applicable instructional response.

Also, the threshold concept character of De Moivre's theorem evidenced by the binary distribution of performance on Question 6, with no partial-credit responses raises an instructional counterfactual worth considering: would a different teaching sequence have produced a different outcome? Specifically, if polar form representation and the geometric interpretation of complex multiplication had been taught and consolidated before De Moivre's theorem was introduced, rather than concurrently or just prior, students might have had the conceptual foundation needed to engage with the theorem productively. The present data cannot answer this question definitively since we do not have a comparison group taught under a different sequence but the complete absence of Procedural or Technical errors in Question 6 suggests that the barrier was conceptual entry, not procedural execution. This implies that additional practice with the theorem's formula would not have helped; what was needed was prior conceptual preparation.

Limitations and Future Directions

Several limitations of this study should inform how its findings are read. First, the sample is small (N = 18) and drawn from a single course at a single institution in Nigeria, limiting the transferability of the findings. The study makes no claim to statistical generalizability; its contribution is interpretive and diagnostic. Second, no follow-up interviews or member-checking procedures were conducted to verify that error categorisations accurately reflect students' actual reasoning. Future studies should address this methodological gap by incorporating student think-aloud protocols or retrospective interviews. Third, the study captures a single moment in time; longitudinal data tracking how these error patterns develop and resolve across a full semester of complex analysis instruction would substantially deepen the picture. Fourth, some the authors of this study are affiliated with the institution at which data were collected while two were involved in teaching MATH307. This proximity carries both an advantage (detailed insider knowledge of the instructional context) and a risk that interpretations may be shaped by prior knowledge of the teaching sequence or individual students. This risk was mitigated through the inter-rater reliability procedure described in the methodology and by grounding all interpretive claims in students' written work as reproduced in the figures; nevertheless, future studies involving independent raters would strengthen the credibility of the categorisation.

Future research should replicate this analysis with larger, multi-site samples across different Nigerian and African university contexts; investigate whether error patterns differ by prior mathematics background, gender, or instructional approach; examine how error types evolve across a full semester of instruction; and explore whether targeted interventions such as task-discrimination activities or GeoGebra-enriched instruction (Seloane et al., 2023) reduce the specific error patterns identified here.

Conclusions and Recommendations

This study set out to characterize the types and patterns of errors made by undergraduate mathematics education students on introductory complex number problems, and to use those errors as windows into the conceptual and procedural gaps in their understanding. The analysis of 18 students' responses to six tasks yields four substantive conclusions.

First, all four error types were present, confirming that students' difficulties are genuinely multidimensional. A single-dimensional account whether 'students don't understand the concepts' or 'students make computational mistakes' would be inadequate. Different tasks elicit different error profiles, and an effective instructional response needs to be equally differentiated.

Second, Interpretation errors were the most consistent cross-task phenomenon. The repeated substitution of modulus-argument and polar-form procedures for simpler tasks observed in 16 instances across Questions 1, 2, and 3 is a class-level pattern with a clear cognitive explanation: students have acquired salient, recently taught schemas and apply them without adequate task discrimination. Instruction must address this directly, through explicit task-discrimination exercises and careful sequencing that allows each procedure to be consolidated before the next is introduced.

Third, Conceptual errors were the most frequent type overall and were overwhelmingly concentrated in Question 6 (De Moivre's theorem). The binary character of performance on this question i.e., full conceptual access for four students, complete inability to begin for twelve signals that De Moivre's theorem functions as a threshold concept requiring dedicated, concept-building instruction. Instructors should not assume that students who can compute moduli and arguments are ready to apply De Moivre's theorem; the theorem requires a qualitatively different level of representational integration.

Fourth, the data reveal a cumulative dependency structure: students who could not divide complex numbers (Question 3) could not simplify the quotient required by Question 5, and students who misunderstood the powers of i (Question 4) lacked the foundation for Question 6. This means that formative assessment of prerequisite skills — rationalization, the behaviour of i^2 must precede instruction on dependent topics.

The principal contribution of this study is the first systematic, figure-grounded error analysis of complex number learning in a Nigerian undergraduate context, one that moves beyond documenting what students cannot do to diagnosing why they cannot do it, and that frames De Moivre's theorem as a threshold concept requiring qualitatively different instructional treatment from other complex number operations.

On the basis of these conclusions, we make six specific recommendations to mathematics instructors (1) Dedicate explicit, extended instructional time to complex number division and rationalization before advancing to modulus, argument, or polar form. (2) Treat $i^2 = -1$ and the cyclical powers of i as foundational identities that require repeated practice and explicit revisiting throughout the course, not just initial introduction. (3) Implement task-discrimination activities i.e., exercises that present students with different complex number tasks side by side and require them to identify the appropriate procedure before executing it to counteract the overgeneralization tendency. (4) Approach De Moivre's theorem through its geometric and conceptual basis (polar representation, the meaning of multiplying complex numbers) rather than as a formula; students need the conceptual journey, not just the destination. (5) Use multimedia resources, including dynamic geometry software and video animations (Caglayan, 2016; Yatab & Shahrill, 2014), and supplementary peer learning sessions (Congos & Schoeps, 1993; McCarthy & Smuts, 1997) to support students in building the multi-representational understanding that complex number competence requires. It is acknowledged that some of these recommendations particularly those involving dynamic geometry software and multimedia resources may face practical constraints in Nigerian university contexts where access to technology and stable internet connectivity cannot be assumed. Instructors working in resource-limited settings may prioritise the task-discrimination activities and sequencing recommendations in (1), (2), and (3), which require no technological infrastructure.

At the policy and curriculum level, these findings suggest that mathematics teacher education programmes in Nigeria and comparable African contexts should explicitly address complex number pedagogy (including the sequencing of topics and the development of task-discrimination skills) as part of pre-service mathematics teacher preparation, rather than assuming that content knowledge alone is sufficient for effective teaching of this topic.

Author Notes

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